EXPLORING PARTITIVE DIVISION WITH YOUNG CHILDREN

Ema Mamede & Amalia Silva

CIEC – University of Minho & J.I. Esposende

This paper focuses on a study with 4- and 5-year-olds children understanding of partitive division when discrete quantities are involved. The study analyse how young children understand the inverse divisor-quotient relationship when the dividend is the same. The participants were 30 kindergarten children from Braga, Portugal. Individual interviews were conducted when solving tasks involving the division of 12 and 24 discrete quantities by 2, 3 and 4 recipients. Results showed that 4- and 5-year-olds children have some ideas of division, can estimate for the quotient when the divisor varies and the dividend is constant, and can justify their answers. Educational implications of these results are discussed for kindergarten activities.

FRAMEWORK

Children learn a considerable amount about mathematical reasoning outside school known as informal knowledge. Literature refers that kindergarten children possess an informal knowledge relevant for many mathematical concepts (see Nunes, 1992; Nunes & Bryant, 1997). This informal knowledge should provide the building of formal mathematical concepts. Concerning the division, several authors suggest that young children can divide discrete quantities successfully (see Frydman & Bryant, 1998; Pepper & Hunting, 1998; Kornilaki & Nunes, 2005; Squire & Bryant, 2002), arguing that these children possess some type of informal knowledge related to the division of quantities, understanding the inverse relation between the divisor and the quotient when the dividend is the same.

Correa, Nunes and Bryant (1998) argue that sharing activities can be relevant in the understating of the inverse relation between the divisor and the quotient. Also Kornilaki and Nunes (2005) argue that understanding the sharing activity helps children to understand the logical relations involved in the division of quantities, i.e., the relation between the dividend, the divisor and the quotient.

When considering the division of discrete quantities it becomes relevant to distinguish the partitive and the quotitive division. In partitive division problem a set of objects is given to be divided among recipients, and the share that each recipient has received is the unknown part. (e.g., there is a set of 10 candies to be shared among 5 children. How many candies does each child get?). In a partitive division problem, the divisor is the number of recipients and the quotient is the share they receive. In quotitive division, there is an initial quantity to be share into a known number of parts. The size of the parts is the unknown (e.g., Mary has 12 candies and wants to give 3 candies to each of her friends. How many friends are receiving the candies?). In quotitive division problems, the divisor is the share to be given to each recipient and the quotient is the number of recipients. Concerning these types of divisions Kornilaki and Nunes (2005) argued that children understand more easily the partitive division than the quotitive division.

Research presents several results of young children procedures when solving division tasks involving discrete quantities (see Piaget & Szieminska, 1971; Desforges & Desforges, 1980; Frydman & Bryant, 1998; Squire & Bryant, 2002). Particularly, Correa, Nunes and Bryant (1998) when investigating the development of the concept of division in young children, examined whether children who could share would be able to understand the inverse divisor-quotient relationship in partitive division tasks when asked to judge the relative size of 2 shared sets. The participants were 20 children of 5-year-olds, 20 of 6-year-olds and 21 of 7-year-olds from Oxford, England. The authors investigated the children's understanding of the three-term quantity relationship in division when the dividend was constant and the divisor varies. In their experiment the experimenter shared a given amount (12 in some trials, 24 in others) of red and blue sweets between two groups of rabbits, one red and one blue, putting the sweets in the boxes attached to the rabbits' backs; the experimenter pointed to one blue rabbit and one red rabbit and each child was asked whether they had the same quantity of sweets or whether one of them received more sweets, and why did the child think so. The authors argued that "if the children succeed in tasks where the dividend is constant and the quotient is inversely related to the divisor, we can be confident that their success indicates some understanding of core relations in a division situations." (p. 322). Results showed that 9 of the 20 5-year-olds performed significantly above chance and about 30% were able to verbalize this inverse relation in their justifications and 11 out of 20 of the 6-year-olds scored above chance and verbalized the inverse relation between the divisor and the quotient in the partitive tasks. The authors also report age improvements between 5 and 7 years. Correa, Nunes and Bryant (1998) also analysed children's justifications according to children's age. Most of the 5-year-olds were not able to give a mathematical justification for their choices and did not mention facts relevant to the solution of the task. The 6-year-olds presented justifications that revealed a progress from some comprehension of sharing and numerical equivalence to the understanding of the inverse divisor-quotient relationship. The majority of the justifications presented by the 7-year-olds showed a logicomathematical approach, referring the inverse divisor-quotient relationship.

More recently, Kornilaki and Nunes (2005) investigated whether the children could transfer their understanding of logical relations from discrete to continuous quantities. Among other things, the authors analysed 32 five-year-olds, 32 six-year-olds and 32 seven-year-olds solving partitive division tasks involving discrete quantities. In this type of problems the number of recipients varied to produce two conditions: 1) in the same divisors condition, the size of the divisor was the same; 2) in the different divisors condition, the number of recipients varied. The results showed that the different divisors condition was clearly more difficult than the same divisors condition. Thus, the authors argued that the inverse relation between the divisor and the quotient is understood later than the equivalence principle of division. The authors also pointed out that in partitive division tasks, one-third of the 5- and 6-year-olds justified their responses as “the more recipients, the more they get”, but this response decreased markedly with age as only slightly more 10% of the 7-year-olds used this incorrect reasoning.

The studies of Correa, Nunes and Bryant (1998) and Kornilaki and Nunes (2005) give evidence that, at age of 6 and 7, children have an insight into relations between the division terms, long before they are introduced to this operation at school. If previous research reports some success with 5-year-olds children, how would children of 4-year-olds would perform? Besides, it becomes relevant to get a better insight on young Portuguese children’s informal knowledge of division.

This paper focuses on young Portuguese children understanding of division of discrete quantities, when solving partitive division problems. For that we tried to address three questions: 1) How do children estimate the quotient in a partitive division in which the divisor varies and the dividend is kept constant? 2) How do children perform the partitive division tasks involving discrete quantities? 3) What procedures do they use in this process?

METHODS

A study focused on young children’s ideas of partitive division was conducted to address these questions. The participants were 15 four-year-olds (11 boys and 4 girls, mean age 4 years and 6 months) and 15 five-year-olds (7 boys and 8 girls, mean age 5 years and 6 months) from Braga, Portugal.

The participants were interviewed individually by one of the researchers when solving the problems. Each problem was presented to each child using a story and manipulatives representing the items involved in each story were available.

Each child was presented to 6 problems: 3 involving the division of 12 units (carrots) by 2, 3 and 4 recipients (rabbits), respectively; and 3 problems involving the division of 24 units (cabbage) by 2, 3 and 4 recipients (rabbits).

In the interview, first children were invited to estimate the effects on the quotient of increasing the divisor keeping the dividend constant. Then they were asked why they thought so. The idea was to have an insight on children’s understanding of the inverse divisor-quotient relationship when the dividend is constant. Then children were asked to carry out the division. In this process, their ability to perform the division was assessed as well as the procedures used by them.

The story presented to the children involved a context in which a white little rabbit had 12 carrots. Then he had to share them fairly with his friend, the brown rabbit. At this moment the child was asked: “Do you think that the white rabbit would be with more or less carrots? Why?”. Them the child was invited to accomplish the division between the two rabbits. Them the child was asked: ”Do you think that both rabbits are happy with this division of the carrots? Why?”, “How many carrots did each received?”. Then a little grey rabbit came around and they had to put all the carrots together again and share them among the three rabbits. “Do you think that each rabbit is going to have more or fewer carrots now?”; “Can you help the rabbits to share the carrots?”; “Do you think that all the rabbits are happy with this division? Why?”. The story continues to include the black rabbit. The same questions were asked. In the very end, when the last rabbit came, the children were asked: “Do you think that all the rabbits are happy with this division? Why? Do you want to check it by counting?”.

When the 24 units were involved, an analogous story was presented to them but now involving the 2, 3 and 4 rabbits and 24 cabbages.

Each child took approximately 20 minutes to solve all the problems, in spite of having no limit for it.

Results

In order to understand children’s ability to estimate the quotient in a partitive division in which the divisor varies and the dividend is kept constant, their correct responses and justifications were analysed. Table 1 resumes the percentage of correct estimates and valid justifications for the division of 12 and 24 units, according to the age. A valid justification is an argument in which a child expresses some ideas of the inverse divisor-quotient relationship, such as “because there are more rabbits and each one get fewer carrots.” or “they will have fewer carrots because now there is the X rabbit”.

	4-year-olds		5-year-olds	
	Correct resp.	Valid argum.	Correct resp.	Valid argum.
12 units	67%	43%	72%	67%
24 units	71%	52%	78%	83%

Table 1: Percentage of correct responses and valid arguments when estimating for the quotient with the dividends of 12 and 24 units, respectively.

It is interesting to note that children's performance in the estimating tasks improved from the first part of the problems (involving 12 units) to the second one (involving 24 units), in spite of the sizes of the initial sets. Perhaps this is due to the fact that when the problems involving the 24 units were presented to the children, they were not a novelty anymore.

Another remarkable point is the success observed among the 4-year-olds when asked to estimate and justify their judgement. Almost half of the children presented a valid justification for their correct answer when dividing the 12 units; when they were asked to divide the 24 units, their valid justifications increased slightly above 50%. These results suggest that children of 4-year-olds may have some ideas about the inverse divisor-quotient relationship presented in these conditions.

Children performance was analysed solving division tasks involving 12 and 24 units by 2, 3 and 4 recipients, respectively. Tables 2 and 3 resume the percentage of children's correct responses by age level, in these problems.

	12 units	
	4-year-olds (n=15)	5-year-olds (n=15)
Division by 2	87%	87%
Division by 3	67%	80%
Division by 4	67%	80%

Table 2: Percentage of correct responses by age level when solving the division of 12 units by 2, 3 and 4 recipients.

	24 units	
	4-year-olds (n=15)	5-year-olds (n=15)
Division by 2	60%	80%
Division by 3	86%	74%
Division by 4	67%	80%

Table 3: Percentage of correct responses by age level when solving the division of 24 units by 2, 3 and 4 recipients.

The results suggest that for young children it becomes more difficult to accomplish the division of 24 units than the division of the 12 units set, possibly due to the magnitude of the set.

As the children’s performance was not normally distributed a Mann-Whitney U Test was conducted in order to analyse children’s performance dividing 12 and 24 units according to the age level. The results show no significant differences on children’s performance when dividing 12 units according to the age levels (age 4, Mdn=3, age 5, Mdn=2, U=149, n.s.) and when dividing 24 units according to the age levels (age 4, Mdn=3, age 5, Mdn=3, U=128, n.s.). Thus, results give evidence that there is no difference of 4- and 5-year-old children’s performance in this division tasks.

Trying to explain these results, children’s procedures were analysed when dividing 12 and 24 units by 2, 3 and 4 recipients, respectively. The same procedures were observed when children were dividing 12 and 24 units. The procedure I comprises the sharing procedures relying on the correspondence one-to-one by the recipients; the procedure II comprises the counting procedures; procedure III comprises sharing activity based on perceptual influence ignoring the size of the shares; and procedure IV comprises sharing activity combined with counting to produce equal shares.

Tables 4 and 5 resume the observed procedures used by the children of both age groups when solving the division problems of 12 and 24 units, respectively.

	12 units							
	4-year-olds (n=15)				5-year-olds (n=15)			
Type of procedure	I	II	III	IV	I	II	III	IV
Division by 2	10	0	3	2	8	2	1	4
Division by 3	9	0	5	1	8	2	3	2
Division by 4	9	1	3	2	8	2	4	1
Total (Max.=45)	28	1	11	5	24	6	8	7

Table 4: Children’s procedures solving the division of 12 units, by age level.

	24 units							
	4-year-olds (n=15)				5-year-olds (n=15)			
Type of procedure	I	II	III	IV	I	II	III	IV
Division by 2	7	0	6	2	9	2	4	0
Division by 3	9	0	5	1	6	2	4	3
Division by 4	9	1	4	1	6	3	4	2
Total (Max.=45)	25	1	15	4	21	7	12	5

Table 5: Children’s procedures solving the division of 24 units, by age level.

The procedures used by children did not change much according to the magnitude of the set to divide. Tables 4 and 5 suggest that sharing assumes an important role on children's performance when solving division problems, with discrete quantities. The sharing activity developed by each child and the type of shares produced give us an insight of children's ideas of fare share. Many 4-year-olds children used sharing activity without recognizing the need of producing fare shares, either when 12 or 24 units were involved (24% and 33%, respectively). This phenomenon was also observed in some 5-years-old children when 12 and 24 units were involved (17.8% and 26.7%, respectively). Nevertheless, the majority of the children of both age groups involved in this study recognized the importance of producing fare shares in the division tasks presented to them.

The procedure mostly used by both age groups of children was correspondence one-to-one. This procedure conducted children to correct resolutions, producing fare shares. The procedures using sharing activity based on perceptual influence ignoring the size of the shares were also popular among children of both age groups.

After carry out the division of the items by the recipients, the children were asked if they were happy with the division made through the question "Do you think that all of the rabbits are happy with this division? Why?". They were also challenged to verify their results by counting - "Do you want to check it by counting?" - to deepen the understanding of children's ideas of fare sharing by giving them an opportunity to correct themselves. Their reactions were analysed and allowed us to distinguished the following categories: CcE comprises children's verifications in which it was observed Correct counting of the items in each recipient when there are already equal shares; CcNon-NE comprises children's verifications in which is was observed Correct counting of the items in each recipient, but without equal shares; NnC comprises children's reactions in which they refuse to verify because they are sure about it and it is correct; NvNE comprise their reactions in which they do not recognise the need to verify and equal shares were not produced; NC comprise children's reaction in which the correct counting of the items was not accomplished.

Tables 6 and 7 resume children's reactions, by age group, when solving the division tasks of 12 and 24 units, respectively. The majority of the children of both age groups used the opportunity to verify their shares, correcting their distributions when necessary. This was observed by 60% of the 4-year-olds and 73.3% of the 5-year-olds when 12 units were involved; and by 51.1% and 62.2% of the 4- and 5-year-olds, respectively, for the 24 units. These results suggest that equal share is a concept understood by young children of 4-year-olds. In most of the problems presented to them, these young children recognised the importance of fair shares when accomplishing a sharing activity in a division of discrete quantities.

	12 units							
	4-year-olds (n=15)				5-year-olds (n=15)			
	Division				Division			
	by 2	by 3	by 4	Total	by 2	by 3	by 4	Total
CcE	9	10	8	27	11	12	11	33
CcNon-NE	2	3	4	9	3	1	3	6
NnC	0	0	0	0	1	1	1	3
NvNE	2	1	1	4	2	1	0	3
NC	2	1	2	5	0	0	0	0

Table 6: Children's reactions to the produced shares after dividing 12 units, by age level.

	24 units							
	4-year-olds (n=15)				5-year-olds (n=15)			
	Division				Division			
	by 2	by 3	by 4	Total	by 2	by 3	by 4	Total
CcE	9	10	8	27	11	12	11	33
CcNon-NE	2	3	4	9	3	1	3	6
NnC	0	0	0	0	1	1	1	3
NvNE	2	1	1	4	2	1	0	3
NC	2	1	2	5	0	0	0	0

Table 7: Children's reactions to the produced shares after dividing 24 units, by age level.

It was also possible to observe a few children who did not need to verify their resolutions that were correct, being sure about their procedures and solutions obtained. A groups of children of both ages did not recognised the need of produce equal shares, in spite of using counting properly when verifying their results (20% and 13.3% of the 4- and 5-year-olds, respectively, when dividing 12 units; and 20% and 35.5% of the 4- and 5-year-olds, respectively, when dividing 24 units).

DISCUSSION AND CONCLUSIONS

The results presented here give some insights of young children ideas of division of discrete quantities but also their ideas of fair sharing. The findings of the study reported here suggest that young children of 4- and 5-year-olds possess some ideas related to the division of quantities, understanding the inverse relation between the divisor and the quotient when the dividend is the same. The analysis conducted here give evidence that children of 4-year-olds reveal some understanding of the effect of increasing the number of recipients when the amount to share is constant. These children were able to estimate the result of division. This suggests that children also have some ideas of the inverse divisor-quotient relationship in partitive division tasks, when asked to judge the relative size of shared sets. This idea is in agreement with Frydman and Bryant (1998), Correa, Nunes and Bryant (1998) and Kornilaki and Nunes (2005).

The study reported here has some similarities with some presented previously in the literature (see Correa, Nunes & Bryant, 1998; Kornilaki & Nunes, 2005) but also offers some original contributions. Correa, Nunes and Bryant (1998) investigated 5- to 7-year-olds children's understanding of inverse divisor-quotient relationship, when partitive division was involved. Their findings give evidence that 5-year-olds children can succeed in these tasks. Also Kornilaki and Nunes (2005) give evidence of 5-year-olds children success when solving this type of tasks. In our study we analysed how children of 4- and 5-year-olds behave when dealing with this type of problems. Some positive signs arise from this investigation. Four-year-olds children are also able to understand some ideas of divisor-quotient relations in particular conditions.

The procedures used by the children of this study suggest that correspondence can play an important role on children's sharing activity and on their accomplishment of division. Some authors argue that sharing activities can be relevant in the understating of the inverse relation between the divisor and the quotient (see Correa, Nunes & Bryant, 1998) and that understanding the sharing activity helps children to understand the relation between the dividend, the divisor and the quotient (see Kornilaki & Nunes, 2005). In agreement with these ideas, one-to-one correspondence sustaining the sharing activity seems to allow young children to understand the logical relations involved in the division of quantities. This study also shows that equal share is a concept understood by some 4-yaer-olds children and recognized by them as an important issue of the division of discrete quantities. Nevertheless, fair sharing does not seem to be only concept for understanding the division of these quantities, as many young children were able to estimate the effects of increasing the divisor in the quotient, for the same dividend, before carry out the division.

These findings suggest that kindergarten activities could stimulate children's early ideas of division, relying of their informal knowledge. These activities could comprise the use of share and the production of equal shares, but also activities to promote the understanding of the logic relations involved in the division, when the dividend is kept constant. These ideas are crucial to understand some complex mathematical concepts such as fractions, later on in the formal traditional school.

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